Matric: 17/ENG02/083

1 If $y = e^{x^2+x}$, show

$y'' = y'(2x+1) + 2y$, & prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)$

solution

$\cancel{y =}$

$\cancel{x^2 \frac{d^2y}{dx^2} + \frac{x\,dy}{dx} + y = 0}$

$y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$u = 2x+1 \quad ; \quad u' = 2$

$v = e^{x^2+x} \quad ; \quad v' = (2x+1)e^{x^2+x}$

$y'' = uv' + vu'$

$= (2x+1)\left[(2x+1)e^{x^2+x}\right] + e^{x^2+x} \times 2$

Let $y' = 2x+1)e^{x^2+x}$, $y = e^{x^2+x}$

$\therefore y'' = y'(2x+1) + \underline{\underline{2y}}$

$y'' - y'(2x+1) - 2y = 0$

Let $A = y''$

$u = u''$ $\quad u^n = u^{n+2}$

$v = 1$ $\quad v' = 0$

$y^n = u^n v + n v^{n-1} v'$

$V = 2 \quad V' = 0$

$y^n = 2y^n \; ; \; ny^{n+1} = 0$

$y^n = 2y^n \Rightarrow C$

$A - B - C = 0$

$A' - B' - C' = 0$

$\therefore y^{n+2} - [y^{n+1}(2x+1) + 2ny^n] - 2y^n = 0$

$y^{n+2} = y^{n+1}(2x+1) + 2ny^n + 2y^n$

$= y^{n+1}(2x+1) + y^n(2n+2)$

$\therefore y^{n+1} = (2x+1)y^{n+1} + 2(n+n)y^n$

21. $y = x^3 e^{4x}$ determine $y^{(5)}$

$u = e^{4x} \; ; \; u^n = 4^n e^{4x}$

$V = x^3 \; ; \; V' = 3x^2, V'' = 6x, V''' = 6, V^{iv} = 0$

$y^n = 4^n e^{4x} \times x^3 + n4^{n-3} e^{4x}(3x^2) + \frac{n(n-1)4^{n-2} e^{4x} + 6x}{2!}$

$+ \frac{n(n-1)(n-2)4^{n-3} e^{4x} - 6}{3!} + C$

$y^n = 4^n e^{4x} x^3 + 3x^2 n \cdot 4^{n-1} e^{4x} + 3x(n^2 - n)4^{n-2} e^{4x}$

11 $x^2 y'' + xy' + y = 0$

$y^n = u^n v + n u^n v' + \frac{n(n-1) u^{(n-2)} v}{2!} + \frac{n(n-1)(n-2) u^{n\,3} v^3}{3!}$ ...

$y = uv$

Let $A = x^3 y''$

$u = y''$ $u^n = y^{n+3}$

$v = x^2$, $v' = 2x$, $v'' = 2$, $v''' = 0$

$y^n = y^{n+2} x^2 + n y^{n+1} \cdot 2x + \frac{n(n-1) y^n \times 2}{2!} + \frac{n(n-1)(n-2) y^{n-1}}{3!}$

$A = y^{n+2} x^2 + n y^{n+1} 2x + n(n-1) y^n$

Let $B = xy'$

$u = y'$, $u^n = y^{n+1}$

$v = x$, $v' = 1$, $v'' = 0$